

Example 4. Set up an iterated integral to find the volume of the solid that is enclosed by the cone $z = \sqrt{x^2 + y^2}$ and the sphere $x^2 + y^2 + z^2 = 2$. Use cylindrical coordinates.

$\iiint_E 1 \, dV$

$x^2 + y^2 + z^2 = 2$
 \Downarrow
 $z = \sqrt{2 - x^2 - y^2} \Rightarrow z = \sqrt{2 - r^2}$

$z = \sqrt{x^2 + y^2} \Rightarrow z = r$

Projection onto xy -plane:

$\sqrt{2 - x^2 - y^2} = \sqrt{x^2 + y^2}$
 $\Rightarrow 2 - x^2 - y^2 = x^2 + y^2$
 $\Rightarrow 2(x^2 + y^2) = 2$
 $\Rightarrow x^2 + y^2 = 1$

$$\iiint_E 1 \, dV = \int_0^{2\pi} \int_0^1 \int_r^{\sqrt{2-r^2}} r \, dz \, dr \, d\theta$$

3 If we have time...

Example 5. Set up an iterated integral to find the volume of the solid above the paraboloid $z = x^2 + y^2$ and below the half-cone $z = \sqrt{x^2 + y^2}$. Use cylindrical coordinates.

$z = x^2 + y^2 \Rightarrow z = r^2$

$z = \sqrt{x^2 + y^2}$
 \Downarrow
 $z = r$

Projection onto xy -plane:

$x^2 + y^2 = \sqrt{x^2 + y^2}$
 $\Rightarrow (x^2 + y^2)^2 = (x^2 + y^2)$
 $\Rightarrow (x^2 + y^2)(x^2 + y^2 - 1) = 0$
 $\Rightarrow x^2 + y^2 = 0$
 $\quad \text{or } \underline{x^2 + y^2 = 1}$

$$\iiint_E 1 \, dV = \int_0^{2\pi} \int_0^1 \int_{r^2}^r r \, dz \, dr \, d\theta$$